Discussion on the paper “Analysis of the dynamic responses for a pre-cracked three-point bend specimen” by Fengchun Jiang, Aashish Rohatgi, Kenneth S. Vecchio and Justin L. Cheney

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The discussed article Fengchun et al. (2004a) is dealing with quite interesting topic. However, it contains several drawbacks that could be misleading for an inexperienced reader.

Theoretical part of the article presents a very simple model of the impact specimen which was proposed by the authors in their previous paper (Fengchun et al., 2004b). As it was shown in previous discussion (Rokach, 2004), accuracy of this model and its applicability for the short time-to-fracture tests is questionable. An example that demonstrates a weak accuracy of the model is presented in the final part of this article.

However the most controversial section of the article under discussion seams to be the fourth one entitled ‘Comparison between the vibration period of the cracked specimen and the apparent period of specimen oscillation’. In this section authors compared the period of apparent specimen oscillations $\tau$ determined by the corresponding formula from the draft ASTM standard (ASTM Draft, 1981) with the natural vibration period of the cracked specimen $T$ determined by author’s specimen model. The authors assumed that $\tau$ should be close to $T$ and discovered that in fact $\tau \approx 0.38 T$. The rest of the section is devoted to a lengthy discussion about the possible reasons of such a large difference between the ‘similar’, as the authors supposed, parameters.

In fact, this discussion is needless, because the initial assumption that $T$ and $\tau$ presents the same physical property is incorrect. Contrary to a bit misleading name, $\tau$ is not related to the natural vibration period of the specimen. As it was clearly shown by Server (1978), $\tau$ is the period of oscillation of a tup force. Thus, it is the period of oscillation of the specimen as a part of the vibrating system ‘specimen + testing machine’. Even for the simplest SDOF model of such a system, $\tau$ can be estimated as $2\pi (m/(k_s + k_m))^{1/2}$, where $m$ is the equivalent mass of the specimen, $k_s$ is its stiffness and $k_m$ is the stiffness of the testing machine (Glover et al., 1977; Williams and Adams, 1987). Usually $k_s << k_m$, so the value of $\tau$ is controlled mainly by $k_m$ and must be essentially lower than $T$ which is close to $2\pi (m/k_s)^{1/2}$.

Authors explained the discrepancy between $\tau$ and $T$ by two new assumptions:

1. Value of $\tau$ in draft standard (ASTM Draft, 1981) is determined for the unsupported specimen in contrast to $T$, that is determined for the simply supported one.
2. Values of $T$ determined for the free-free impact specimen can be several times smaller than the similar values determined for the simply supported one.

Unfortunately, both these assumptions are again incorrect. The first one contradicts the precise definition of $\tau$ as 'a time between the $2\tau$ and $3\tau$' (Server, 1978) (means time interval between 2nd and 3rd minima of the force–time diagram). Some authors suggested to measure $\tau$ values even later – somewhere between $3\tau$ and $5\tau$ (Ireland, 1977). This definition was made intentionally to avoid determination of $\tau$ during the initial stage of the test when specimen boundary conditions are unstable. Thus, regardless of the precise meaning of this parameter, $\tau$ is definitely determined for the established three-point-bending boundary conditions only.

Second assumption is incorrect because the difference between the natural eigenfrequencies of the free–free ($\omega_{f1}$) and simply supported ($\omega_{s1}$) impact specimen considered in Fengchun et al. (2004a) is not as high as suggested by the authors. Just the opposite, these two eigenfrequencies are usually very close for the specimens with relatively long overhangs. Authors had a chance to calculate $\omega_{f1}$ for their specimen very simply using the following formula proposed in Rokach (1998) (this paper was even cited in the discussed article)

$$\omega_{f1} = \frac{1}{W} \sqrt{\frac{E}{\rho}} \left( -0.2346 + 0.38\lambda - 0.16\lambda^3 + \frac{W}{L} (2.5 - 3.12\lambda + 0.59\lambda^3) \right)$$

(1)

where $\lambda$ is the relative crack length, $L$, $W$ are the specimen length and width, respectively, $E$ and $\rho$ are Young modulus and material density, respectively. This formula fits the results of plane stress finite element analysis (FEA) with accuracy about 1% for $\lambda = 0.3 - 0.7$, $L/W = 4 - 6$. Instead, authors used numerical results obtained in Sahraou and Latallade (1998) for the system ‘specimen + testing machine’ and, therefore, quite different from the same for the specimen itself.

In addition to Eq. (1), to find the difference between $\omega_{f1}$ and $\omega_{s1}$ precisely and to check the accuracy of the model presented in the discussed paper, these values have been determined by FEA. Calculations have been performed both for 2D (plane stress and plane strain) and 3D models of the specimen used in Fengchun et al. (2004a) ($L = 40$ mm, $W = 6.5$ mm, specimen thickness $B = 4$ mm, $\lambda = 0.5$, $E = 210$ GPa, $v = 0.25$, $\rho = 7800$ kg/m$^3$) using the commercial program ADINA 8.1.2. Results of these calculations (see Table 1) show that:

<table>
<thead>
<tr>
<th>$\omega_{f1}$, $\times 10^4$ rad/s</th>
<th>FEA, 3D</th>
<th>FEA, pl. stress</th>
<th>FEA, pl. strain</th>
<th>Eq. (1), pl. stress</th>
<th>Fengchun et al. (2004a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_{f1}$, $\times 10^4$ rad/s</td>
<td>7.38</td>
<td>7.34</td>
<td>7.57</td>
<td>8.99</td>
<td></td>
</tr>
<tr>
<td>Difference, %</td>
<td>-</td>
<td>-0.6</td>
<td>2.5</td>
<td>21.8</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Comparison of the natural eigenfrequency of the specimen determined by FEA, Eq. (1) and by the model proposed in Fengchun et al. (2004a). Differences are with respect to the most accurate (i.e. 3D) results.
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1. The difference between $\omega^f_1$ and $\omega^s_1$ is only about 7.5%.
2. Contrary to another assumption used in Fengchun et al. (2004a), results obtained using the plane stress model of the specimen are much closer to the 3D data than the plane strain results.
3. The model proposed in Fengchun et al. (2004a) overestimates $\omega^s_1$ essentially.

References